実時間法〜ϕ^3 理論〜

実時間法の摂動展開を ϕ^3 理論で行います。ここでは単に摂動展開の計算をしているだけなので、興味がない人は飛ばしていいです。ϕ^3 理論のファインマン則のまとめは「実時間法〜ϕ^4 理論〜」の最後に載っています。
記号とかの定義は「実時間法〜ϕ^4 理論〜」と同じになっています。

「実時間法〜ϕ^4 理論〜」と同じように実数スカラー場での相互作用なしの生成汎関数は

$$
\begin{aligned}
Z_0[J] =& \exp\Big[-\frac{i}{2}\int d^4x\int d^4y\ J_a(x)\Delta_{(ab)}(x,y)J_b(y)\Big] \\
=& \exp\Big[-\frac{i}{2}\int d^4x\int d^4y\ J_1(x)\Delta_{(11)}(x,y)J_1(y) - \frac{i}{2}\int d^4x\int d^4y\ J_2(x)\Delta_{(22)}(x,y)J_2(y) \\
&\quad -\frac{i}{2}\int d^4x\int d^4y\ J_1(x)\Delta_{(12)}(x,y)J_2(y) - \frac{i}{2}\int d^4x\int d^4y\ J_2(x)\Delta_{(21)}(x,y)J_1(y)\Big]
\end{aligned}
$$

ローマ文字の添え字は $1, 2$ で、同じ添え字に関しては和を取らせ、J による汎関数微分は

$$
\frac{\delta J_a(x')}{\delta J_b(x)} = \delta_{ab}\delta^4(x'-x)
$$

と定義します。そして、場と源との関係は

$$
\phi_1(x)\ \Leftrightarrow\ \frac{1}{i}\frac{\delta}{\delta J_1(x)}\ ,\ \phi_2(x)\ \Leftrightarrow\ \frac{1}{i}\frac{\delta}{\delta J_2(x)}
$$

相互作用ありだとして、摂動展開したときの生成汎関数は

$$
\begin{aligned}
Z[J] =& \Big(1+\Big(i\int d^4x\mathcal{L}_{int}[\frac{1}{i}\frac{\delta}{\delta J_1}] - i\int d^4x\mathcal{L}_{int}[\frac{1}{i}\frac{\delta}{\delta J_2}]\Big)+\cdots\Big) \\
&\times\exp\Big[-\frac{i}{2}\int d^4x\int d^4y\ J_a(x)\Delta_{(ab)}(x,y)J_b(y)\Big]
\end{aligned}
$$

J_a による exp 部分の汎関数微分は

$$\frac{\delta}{\delta J_1(z)}\exp[\quad] = M_1(z;J)\exp[\quad]$$

$$\frac{\delta M_1(z_1;J)}{\delta J_1(z_2)} = -\frac{1}{2}D_{(11)}(z_1,z_2) - \frac{1}{2}D_{(11)}(z_2,z_1) = -D_{(11)}(z_1,z_2)$$

$$\frac{\delta}{\delta J_2(z)}\exp[\quad] = M_2(z;J)\exp[\quad]$$

$$\frac{\delta M_2(z_1;J)}{\delta J_2(z_2)} = -D_{(22)}(z_1,z_2)$$

$$\frac{\delta M_1(z_1;J)}{\delta J_2(z_2)} = -D_{(12)}(z_1,z_2)$$

$$\frac{\delta M_2(z_1;J)}{\delta J_1(z_2)} = -D_{(21)}(z_1,z_2)$$

ϕ^3 理論は

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3$$

となっていて、摂動展開したときの寄与は 2 次のオーダからなので

$$Z_2[J] = \frac{1}{2}\Big(-\frac{i\lambda}{3!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J_1(z)}\right)^3 + \frac{i\lambda}{3!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J_2(z)}\right)^3\Big)^2\exp\Big[-\frac{1}{2}\int d^4xd^4y\ J_a(x)D_{ab}(x,y)J_b(y)\Big]$$

となります。なので

$$\begin{aligned}
&\Big(-\frac{i\lambda}{3!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J_1(z)}\right)^3 + \frac{i\lambda}{3!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J_2(z)}\right)^3\Big)^2\\
&= (\frac{i\lambda}{3!})^2\int d^4z_1d^4z_2(\frac{1}{i}\frac{\delta}{\delta J_1(z_1)})^3(\frac{1}{i}\frac{\delta}{\delta J_1(z_2)})^3 + (\frac{i\lambda}{3!})^2\int d^4z_1d^4z_2(\frac{1}{i}\frac{\delta}{\delta J_2(z_1)})^3(\frac{1}{i}\frac{\delta}{\delta J_2(z_2)})^3\\
&\quad - 2(\frac{i\lambda}{3!})^2\int d^4z_1d^4z_2(\frac{1}{i}\frac{\delta}{\delta J_1(z_1)})^3(\frac{1}{i}\frac{\delta}{\delta J_2(z_2)})^3
\end{aligned}$$

この 3 つの項による汎関数微分を計算していきます。見てわかるように今度は J_1 と J_2 が相当混ざってきます。
第一項は J_1 しかいないので単純に計算していけばよくて

$$\begin{aligned}
I_1 = &\int d^4z_1d^4z_2(\frac{1}{i}\frac{\delta}{\delta J_1(z_1)})^3(\frac{1}{i}\frac{\delta}{\delta J_1(z_2)})^3\exp\Big[-\frac{1}{2}\int d^4xd^4y\ J_a(x)D_{ab}(x-y)J_b(y)\Big]\\
&= \frac{1}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^3(\frac{\delta}{\delta J_1(z_2)})^2M_1(z_2;J)\exp[\quad]\\
&= \frac{1}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^3\frac{\delta}{\delta J_1(z_2)}(-D_{(11)}(z_2,z_2) + M_1^2(z_2;J))\exp[\quad]\\
&= \frac{1}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^3(-3D_{(11)}(z_2,z_2)M_1(z_2;J) + M_1^3(z_2;J))\exp[\quad]
\end{aligned}$$

ここから z_2 でなく z_1 になってることに注意してください。

$$
\begin{aligned}
I_1 = &\frac{1}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^2\big(3D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)-3D_{(11)}(z_2,z_2)M_1(z_1;J)M_1(z_2;J)\\
&-3D_{(11)}(z_2,z_1)M_1^2(z_2;J)+M_1(z_1;J)M_1^3(z_2;J)\big)\exp[\quad]\\
=&\frac{1}{i^6}\int d^4z_1d^4z_2\frac{\delta}{\delta J_1(z_1)}\big(3D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1(z_1;J)\\
&+3D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_2;J)+3D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1(z_1;J)\\
&-3D_{(11)}(z_2,z_2)M_1^2(z_1;J)M_1(z_2;J)+6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(z_2;J)\\
&-3D_{(11)}(z_2,z_1)M_1(z_1;J)M_1^2(z_2;J)-D_{(11)}(z_1,z_1)M_1^3(z_2;J)\\
&-3D_{(11)}(z_2,z_1)M_1(z_1;J)M_1^2(z_2;J)+M_1^2(z_1;J)M_1^3(z_2;J)\big)\exp[\quad]\\
=&\frac{1}{i^6}\int d^4z_1d^4z_2\frac{\delta}{\delta J_1(z_1)}\big(6D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1(z_1;J)\\
&+3D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_2;J)+6D_{11}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(z_2;J)\\
&-3D_{(11)}(z_2,z_2)M_1^2(z_1;J)M_1(z_2;J)-6D_{(11)}(z_2,z_1)M_1(z_1;J)M_1^2(z_2;J)\\
&-D_{(11)}(z_1,z_1)M_1^3(z_2;J)+M_1^2(z_1;J)M_1^3(z_2;J)\big)\exp[\quad]\\
=&\frac{1}{i^6}\int d^4z_1d^4z_2\big(-6D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,z_1)+6D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1^2(z_1;J)\\
&-3D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(11)}(z_2,z_1)+3D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_1(z_2;J)\\
&-6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)+6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(z_1;J)M_1(z_2;J)\\
&+6D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_1(z_2;J)+3D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1^2(z_1;J)\\
&-3D_{(11)}(z_2,z_2)M_1^3(z_1;J)M_1(z_2;J)+6D_{(11)}(z_2,z_1)D_{(11)}(z_1,z_1)M_1^2(z_2;J)\\
&+12D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(z_1;J)M_1(z_2;J)-6D_{(11)}(z_2,z_1)M_1^2(z_1;J)M_1^2(z_2;J)\\
&+3D_{(11)}(z_1,z_1)D_{(11)}(z_2,z_1)M_1^2(z_2;J)-D_{(11)}(z_1,z_1)M_1(z_1;J)M_1^3(z_2;J)\\
&-2D_{(11)}(z_1,z_1)M_1(z_1;J)M_1^3(z_2;J)-3D_{(11)}(z_2,z_1)M_1^2(z_1;J)M_1^2(z_2;J)\\
&+M_1^3(z_1;J)M_1^3(z_2;J)\big)\exp[\quad]\\
=&\frac{1}{i^6}\int d^4z_1d^4z_2\big(-9D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,z_1)-6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)\\
&+18D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)M_1^2(z_1;J)+9D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_1(z_2;J)\\
&+18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(z_1;J)M_1(z_2;J)-3D_{(11)}(z_2,z_2)M_1^3(z_1;J)M_1(z_2;J)\\
&-9D_{(11)}(z_2,z_1)M_1^2(z_1;J)M_1^2(z_2;J)-3D_{(11)}(z_1,z_1)M_1(z_1;J)M_1^3(z_2;J)\\
&+M_1^3(z_1;J)M_1^3(z_2;J)\big)\exp[\quad]
\end{aligned}
$$

J_2 での場合はこれの C_1 上であることを表す添え字 1 を 2 に変えればいいだけで

$$
\begin{aligned}
I_2 = \frac{1}{i^6}\int d^4z_1 d^4z_2 \big(& -9D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(22)}(z_1,z_1) - 6D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1) \\
& + 18D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)M_2^2(z_1;J) + 9D_{(22)}(z_2,z_2)D_{(22)}(z_1,z_1)M_2(z_1;J)M_2(z_2;J) \\
& + 18D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)M_2(z_1;J)M_2(z_2;J) - 3D_{(22)}(z_2,z_2)M_2^3(z_1;J)M_2(z_2;J) \\
& - 9D_{(22)}(z_2,z_1)M_2^2(z_1;J)M_2^2(z_2;J) - 3D_{(22)}(z_1,z_1)M_2(z_1;J)M_2^3(z_2;J) \\
& + M_2^3(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]
\end{aligned}
$$

最後の J_1 と J_2 の両方がある項は (この場合では z_1 が C_1 上、z_2 が C_2 上にいます)

$$
\begin{aligned}
I_3 =& -2\int d^4z_1d^4z_2(\frac{1}{i}\frac{\delta}{\delta J_1(z_1)})^3(\frac{1}{i}\frac{\delta}{\delta J_2(z_2)})^3\exp[\quad]\\
=& -\frac{2}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^3(-3D_{(22)}(z_2,z_2)M_2(z_2;J)+M_2^3(z_2;J))\exp[\quad]\\
=& -2\frac{1}{i^6}\int d^4z_1d^4z_2(\frac{\delta}{\delta J_1(z_1)})^2\big(3D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)-3D_{(22)}(z_2,z_2)M_1(z_1;J)M_2(z_2;J)\\
&-3D_{(21)}(z_2,z_1)M_2^2(z_2;J)+M_1(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]\\
=& -\frac{2}{i^6}\int d^4z_1d^4z_2\frac{\delta}{\delta J_1(z_1)}\big(3D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1(z_1;J)\\
&+3D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)M_2(z_2;J)+3D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1(z_1;J)\\
&-3D_{(22)}(z_2,z_2)M_1^2(z_1;J)M_2(z_2;J)+6D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)M_2(z_2;J)\\
&-3D_{(21)}(z_2,z_1)M_1(z_1;J)M_2^2(z_2;J)-D_{(11)}(z_1,z_1)M_2^3(z_2;J)\\
&-3D_{(21)}(z_2,z_1)M_1(z_1;J)M_2^2(z_2;J)+M_1^2(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]\\
=& -\frac{2}{i^6}\int d^4z_1d^4z_2\frac{\delta}{\delta J_1(z_1)}\big(6D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1(z_1;J)\\
&+3D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)M_2(z_2;J)+6D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)M_2(z_2;J)\\
&-3D_{(22)}(z_2,z_2)M_1^2(z_1;J)M_2(z_2;J)-6D_{(21)}(z_2,z_1)M_1(z_1;J)M_2^2(z_2;J)\\
&-D_{(11)}(z_1,z_1)M_2^3(z_2;J)+M_1^2(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]\\
=& -\frac{2}{i^6}\int d^4z_1d^4z_2\big(-6D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,z_1)+6D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1^2(z_1;J)\\
&-3D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)+3D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_2(z_2;J)\\
&-6D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)+6D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)M_1(z_1;J)M_2(z_2;J)\\
&+6D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_2(z_2;J)+3D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1^2(z_1;J)\\
&-3D_{(22)}(z_2,z_2)M_1^3(z_1;J)M_2(z_2;J)+6D_{(21)}(z_2,z_1)D_{(11)}(z_1,z_1)M_2^2(z_2;J)\\
&+12D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)M_1(z_1;J)M_2(z_2;J)-6D_{(21)}(z_2,z_1)M_1^2(z_1;J)M_2^2(z_2;J)\\
&+3D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)M_2^2(z_2;J)-D_{(11)}(z_1,z_1)M_1(z_1;J)M_2^3(z_2;J)\\
&-2D_{(11)}(z_1,z_1)M_1(z_1;J)M_2^3(z_2;J)-3D_{(21)}(z_2,z_1)M_1^2(z_1;J)M_2^2(z_2;J)\\
&+M_1^3(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]\\
=& -\frac{2}{i^6}\int d^4z_1d^4z_2\big(-9D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)-6D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)\\
&+9D_{(22)}(z_2,z_2)D_{(11)}(z_1,z_1)M_1(z_1;J)M_2(z_2;J)+18D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)M_1(z_1;J)M_2(z_2;J)\\
&+9D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)M_1^2(z_1;J)+9D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)M_2^2(z_2;J)\\
&-3D_{(22)}(z_2,z_2)M_1^3(z_1;J)M_2(z_2;J)-3D_{(11)}(z_1,z_1)M_1(z_1;J)M_2^3(z_2;J)\\
&-9D_{(21)}(z_2,z_1)M_1^2(z_1;J)M_2^2(z_2;J)+M_1^3(z_1;J)M_2^3(z_2;J)\big)\exp[\quad]
\end{aligned}
$$

これら 3 つの部分を足したものが 2 次のオーダからの寄与

$$Z_2[J]=\frac{1}{2}\big((\frac{i\lambda}{3!})^2I_1+(\frac{i\lambda}{3!})^2I_2+(\frac{i\lambda}{3!})^2I_3\big)$$

となります。

伝播関数への寄与をみたいので、$J_1=J_2=0$ で明らかに落ちそうな項を無視して汎関数微分を実行すれば、

$$\begin{aligned}
K_1(x,y)&=(\frac{i\lambda}{3!})^2\frac{1}{i^2}\frac{\delta^2 I_1}{\delta J_1(x)\delta J_1(y)}|_{J_1=J_2=0}\\
&=-(\frac{i\lambda}{3!})^2\frac{1}{i^2}\frac{\delta}{\delta J_1(x)}\int d^4z_1d^4z_2\big(-9D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,z_1)M_1(y;J)\\
&\quad-6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)M_1(y;J)\\
&\quad-36D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)M_1(z_1;J)\\
&\quad-9D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(11)}(z_1,y)M_1(z_2;J)\\
&\quad-9D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(11)}(z_2,y)M_1(z_1;J)\\
&\quad-18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)M_1(z_2;J)\\
&\quad-18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,y)M_1(z_1;J)\big)\exp[\quad]|_{J_1=J_2=0}\\
&=(\frac{i\lambda}{3!})^2\int d^4z_1d^4z_2\big(9D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,z_1)D_{(11)}(y,x)\\
&\quad+6D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(y,x)\\
&\quad+36D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_1,x)\\
&\quad+9D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(11)}(z_1,y)D_{(11)}(z_2,x)\\
&\quad+9D_{(11)}(z_2,z_2)D_{(11)}(z_1,z_1)D_{(11)}(z_2,y)D_{(11)}(z_1,x)\\
&\quad+18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_2,x)\\
&\quad+18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,y)D_{(11)}(z_1,x)\big)
\end{aligned}$$

このとき disconnected でないものは

$$
\begin{aligned}
K_1(x,y) = (\frac{i\lambda}{3!})^2 \int d^4z_1 d^4z_2 \\
&\times \big(36D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_1,x) \\
&\quad + 18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_2,x) \\
&\quad + 18D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_2,y)D_{(11)}(z_1,x)\big) \\
= (i\lambda)^2 \int d^4z_1 d^4z_2 \\
&\times \big(D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_1,x) \\
&\quad + D_{(11)}(z_2,z_1)D_{(11)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_2,x)\big) \\
= (i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(11)}(y,z_1)D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,x) \\
&\quad + D_{(11)}(y,z_1)D_{(11)}(z_1,z_2)D_{(11)}(z_1,z_2)D_{(11)}(z_2,x)\big)
\end{aligned}
$$

右から $x \to y$ へと追っていくように並べなおしています。
I_2 でも disconnected な項を無視することで

$$
\begin{aligned}
K_2(x,y) &= (\frac{i\lambda}{3!})^2 \frac{1}{i^2} \frac{\delta^2 I_2}{\delta J_1(x)\delta J_1(y)}|_{J_1=J_2=0} \\
&= (\frac{i\lambda}{3!})^2 \frac{\delta}{\delta J_1(x)} \int d^4z_1 d^4z_2 \big(-36D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)M_2(z_1;J) \\
&\qquad - 18D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)M_2(z_2;J) \\
&\qquad - 18D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(21)}(z_2,y)M_2(z_1;J)\big)\exp[\quad]|_{J_1=J_2=0} \\
&= (\frac{i\lambda}{3!})^2 \int d^4z_1 d^4z_2 \big(36D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)D_{(21)}(z_1,x) \\
&\qquad + 18D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)D_{(21)}(z_2,x) \\
&\qquad + 18D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(21)}(z_2,y)D_{(21)}(z_1,x)\big)\exp[\quad]|_{J_1=J_2=0} \\
&= (i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)D_{(21)}(z_1,x) \\
&\qquad + D_{(22)}(z_2,z_1)D_{(22)}(z_2,z_1)D_{(21)}(z_1,y)D_{(21)}(z_2,x)\big) \\
&= (i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(12)}(y,z_1)D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(21)}(z_1,x) \\
&\qquad + D_{(12)}(y,z_1)D_{(22)}(z_1,z_2)D_{(22)}(z_1,z_2)D_{(21)}(z_2,x)\big)
\end{aligned}
$$

$D_{(12)}(v,w) = D_{(21)}(w,v)$ となっていることに注意してください (x, y は C_1 上、z_1, z_2 は C_2 上)。
I_3 では

$$K_3(x,y) = (\frac{i\lambda}{3!})^2 \frac{1}{i^2} \frac{\delta^2 I_3}{\delta J_1(x)\delta J_2(y)}|_{J_1=J_2=0}$$

$$= -2(\frac{i\lambda}{3!})^2 \frac{\delta}{\delta J_1(x)} \int d^4z_1 d^4z_2 \big(-18D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,y)M_1(z_1;J)$$

$$-18D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,y)M_2(z_2;J)$$

$$-18D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,y)M_2(z_2;J)$$

$$-18D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,y)M_1(z_1;J)\big) \exp[\quad]|_{J_1=J_2=0}$$

$$= -(i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,y)D_{(11)}(z_1,x)$$

$$+ D_{(11)}(z_1,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,y)D_{(21)}(z_2,x)$$

$$+ D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,y)D_{(21)}(z_2,x)$$

$$+ D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(21)}(z_2,y)D_{(11)}(z_1,x)\big)$$

$$= -(i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(11)}(y,z_1)D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)$$

$$+ D_{(12)}(y,z_2)D_{(11)}(z_1,z_1)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)$$

$$+ D_{(11)}(y,z_1)D_{(12)}(z_1,z_2)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)$$

$$+ D_{(12)}(y,z_2)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)\big)$$

というわけで、2 次のオーダからの寄与は

$$\frac{1}{2}(K_1(x,y)+K_2(x,y)+K_3(x,y))$$

$$= \frac{1}{2}(i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(11)}(y,z_1)D_{(11)}(z_2,z_2)D_{(11)}(z_2,z_1)D_{(11)}(z_1,x)$$

$$+ D_{(11)}(y,z_1)D_{(11)}(z_1,z_2)D_{(11)}(z_1,z_2)D_{(11)}(z_2,x)\big)$$

$$+ \frac{1}{2}(i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(12)}(y,z_1)D_{(22)}(z_2,z_2)D_{(22)}(z_2,z_1)D_{(21)}(z_1,x)$$

$$+ D_{(12)}(y,z_1)D_{(22)}(z_1,z_2)D_{(22)}(z_1,z_2)D_{(21)}(z_2,x)\big)$$

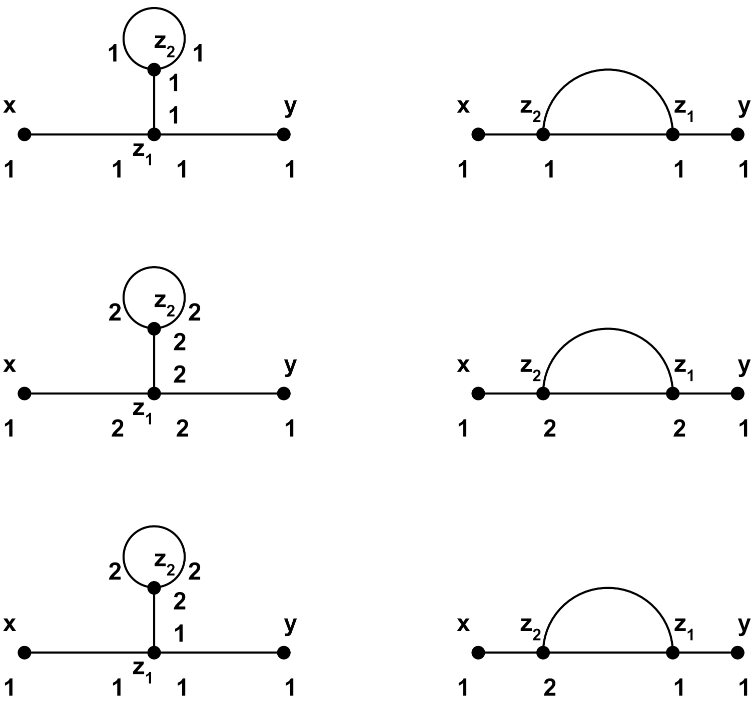
$$- \frac{1}{2}(i\lambda)^2 \int d^4z_1 d^4z_2 \big(D_{(11)}(y,z_1)D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)$$

$$+ D_{(12)}(y,z_2)D_{(11)}(z_1,z_1)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)$$

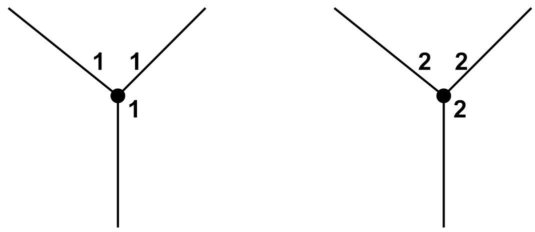
$$+ D_{(11)}(y,z_1)D_{(12)}(z_1,z_2)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)$$

$$+ D_{(12)}(y,z_2)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)\big)$$

この場合でも基本となる図はゼロ温度と同じ 2 つの図で、今の場合では



一行目の図は K_1、2 行目の図は K_2、3 行目の図は K_3 の第一項と第四項で、K_3 での第二項と第三項は 3 行目の 2 つの図で z_1 と z_2 を逆にしたものです (K_3 では、z_1 は C_1、z_2 は C_2 にいるので、伝播関数の添え字も 1 を 2 に、2 を 1 に入れ替える)。頂点が



という 2 種類 (頂点に入ってくる線が $1,1,1$ での $-i\lambda$ と、$2,2,2$ での $+i\lambda$) があるために 8 つの図が現れます (頂点が 2 種類なので $2\times 2=4$ で、2 つの図の形があるので $4\times 2=8$)。ちなみに $1/2$ はファインマン則での対称因子に対応します。K_3 部分の符号がマイナスになっているのはひとつの図に $1,1,1$ の頂点と $2,2,2$ の頂点があるために、$(-i\lambda)(+i\lambda)$ となっているからです。

運動量表示では

$$
\begin{aligned}
K_1(p_1,p_2) =& \int d^4x d^4y K_1(x,y) e^{ip_1x} e^{ip_2y} \\
=& (i\lambda)^2 \int d^4x d^4y d^4z_1 d^4z_2 \int \frac{d^4q_1\cdots d^4q_4}{(2\pi)^{16}} D_{(11)}(q_1)D_{(11)}(q_2)D_{(11)}(q_3)D_{(11)}(q_4) \\
&\qquad \times e^{ip_1x}e^{ip_2y}e^{-iq_1(y-z_1)}e^{-iq_3(z_2-z_1)}e^{-iq_4(z_1-x)} \\
&+ (i\lambda)^2 \int d^4x d^4y d^4z_1 d^4z_2 \int \frac{d^4q_1\cdots d^4q_4}{(2\pi)^{16}} D_{(11)}(q_1)D_{(11)}(q_2)D_{(11)}(q_3)D_{(11)}(q_4) \\
&\qquad \times e^{ip_1x}e^{ip_2y}e^{-iq_1(y-z_1)}e^{-iq_2(z_1-z_2)}e^{-iq_3(z_1-z_2)}e^{-iq_4(z_2-x)} \\
=& (i\lambda)^2 \int d^4q_1\cdots d^4q_4 D_{(11)}(q_1)D_{(11)}(q_2)D_{(11)}(q_3)D_{(11)}(q_4) \\
&\qquad \times \delta^4(p_1+q_4)\delta^4(p_2-q_1)\delta^4(q_3)\delta^4(q_1+q_3-q_4) \\
&+ (i\lambda)^2 \int d^4q_1\cdots d^4q_4 D_{(11)}(q_1)D_{(11)}(q_2)D_{(11)}(q_3)D_{(11)}(q_4) \\
&\qquad \times \delta^4(p_1+q_4)\delta^4(p_2-q_1)\delta^4(q_1-q_2-q_3)\delta^4(q_2+q_3-q_4) \\
=& (i\lambda)^2 \int d^4q_2 D_{(11)}(p_2)D_{(11)}(q_2)D_{(11)}(-p_1-p_2)D_{(11)}(-p_1)\delta^4(-p_1-p_2) \\
&+ (i\lambda)^2 \int d^4q_2 D_{(11)}(p_2)D_{(11)}(q_2)D_{(11)}(p_2-q_2)D_{(11)}(-p_1)\delta^4(p_1+p_2)
\end{aligned}
$$

$-p_1=p_2=p$ としてデルタ関数をはずせば

$$
\begin{aligned}
K_1(p) =& (i\lambda)^2 \int \frac{d^4q}{(2\pi)^4} D_{(11)}(p)D_{(11)}(q)D_{(11)}(0)D_{(11)}(p) \\
&+ (i\lambda)^2 \int \frac{d^4q}{(2\pi)^4} D_{(11)}(p)D_{(11)}(q)D_{(11)}(p-q)D_{(11)}(p) \\
K_2(p) =& (i\lambda)^2 \int \frac{d^4q}{(2\pi)^4} D_{(12)}(p)D_{(22)}(q)D_{(22)}(0)D_{(21)}(p) \\
&+ (i\lambda)^2 \int \frac{d^4q}{(2\pi)^4} D_{(12)}(p)D_{(22)}(q)D_{(22)}(p-q)D_{(21)}(p)
\end{aligned}
$$

K_3 は少し変更されるので一応まじめにやっておきます。

$$
\begin{aligned}
K_3(p_1,p_2) =& \int d^4xd^4yK_3(x,y)e^{ip_1x}e^{ip_2y} \\
=& -(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(11)}(y,z_1)D_{(22)}(z_2,z_2)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)e^{ip_1x}e^{ip_2y} \\
&-(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(12)}(y,z_2)D_{(11)}(z_1,z_1)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)e^{ip_1x}e^{ip_2y} \\
&-(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(11)}(y,z_1)D_{(12)}(z_1,z_2)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)e^{ip_1x}e^{ip_2y} \\
&-(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(12)}(y,z_2)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)e^{ip_1x}e^{ip_2y}
\end{aligned}
$$

第一項と第三項は K_1,K_2 と同じで、第二項と第四項では z_1 と z_2 が一部入れ替わっています。なので、この部分だけ変更します。そうすると第二項は

$$
\begin{aligned}
&-(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(12)}(y,z_2)D_{(11)}(z_1,z_1)D_{(12)}(z_1,z_2)D_{(21)}(z_2,x)e^{ip_1x}e^{ip_2y} \\
&= -(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2\int \frac{d^4q_1\cdots d^4q_4}{(2\pi)^{16}}D_{(12)}(q_1)D_{(11)}(q_2)D_{(12)}(q_3)D_{(21)}(q_4) \\
&\qquad\qquad \times e^{ip_1x}e^{ip_2y}e^{-iq_1(y-z_2)}e^{-iq_3(z_1-z_2)}e^{-iq_4(z_2-x)} \\
&= -(i\lambda)^2\int d^4q_1\cdots d^4q_4D_{(12)}(q_1)D_{(11)}(q_2)D_{(12)}(q_3)D_{(21)}(q_4) \\
&\qquad\qquad \times \delta^4(p_1+q_4)\delta^4(p_2-q_1)\delta^4(q_3)\delta^4(q_1+q_3-q_4) \\
&= -(i\lambda)^2\int d^4q_2D_{(12)}(p_2)D_{(11)}(q_2)D_{(12)}(-p_1-p_2)D_{(21)}(-p_1)\delta^4(-p_1-p_2) \\
&= -(i\lambda)^2\int \frac{d^4q_2}{(2\pi)^4}D_{(12)}(p_2)D_{(11)}(q_2)D_{(12)}(-p_1-p_2)D_{(21)}(-p_1)(2\pi)^4\delta^4(p_1+p_2) \\
&\Rightarrow -(i\lambda)^2\int \frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(11)}(q)D_{(12)}(0)D_{(21)}(p)
\end{aligned}
$$

第四項は

$$
\begin{aligned}
&-(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2D_{(12)}(y,z_2)D_{(21)}(z_2,z_1)D_{(21)}(z_2,z_1)D_{(11)}(z_1,x)e^{ip_1x}e^{ip_2y}\\
&=\ -(i\lambda)^2\int d^4xd^4yd^4z_1d^4z_2\int\frac{d^4q_1\cdots d^4q_4}{(2\pi)^{16}}D_{(12)}(q_1)D_{(21)}(q_2)D_{(21)}(q_3)D_{(11)}(q_4)\\
&\qquad\qquad\qquad\times e^{ip_1x}e^{ip_2y}e^{-iq_1(y-z_2)}e^{-iq_2(z_2-z_1)}e^{-iq_3(z_2-z_1)}e^{-iq_4(z_1-x)}\\
&=\ -(i\lambda)^2\int d^4q_1\cdots d^4q_4D_{(12)}(q_1)D_{(21)}(q_2)D_{(21)}(q_3)D_{(11)}(q_4)\\
&\qquad\qquad\qquad\times\delta^4(p_1+q_4)\delta^4(p_2-q_1)\delta^4(q_2+q_3-q_4)\delta^4(q_1-q_2-q_3)\\
&=\ -(i\lambda)^2\int d^4q_2D_{(12)}(p_2)D_{(21)}(q_2)D_{(21)}(-p_1-q_2)D_{(11)}(-p_1)\delta^4(p_1+p_2)\\
&=\ -(i\lambda)^2\int\frac{d^4q_2}{(2\pi)^4}D_{(12)}(p_2)D_{(21)}(q_2)D_{(21)}(-p_1-q_2)D_{(11)}(-p_1)(2\pi)^4\delta^4(p_1+p_2)\\
&\Rightarrow\ -(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(21)}(q)D_{(21)}(p-q)D_{(11)}(p)
\end{aligned}
$$

よって

$$
\begin{aligned}
K_3(p)=\ &-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(11)}(q)D_{(12)}(0)D_{(21)}(p)\\
&-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(11)}(q)D_{(12)}(0)D_{(21)}(p)\\
&-(i\lambda)^2\int\frac{d^4q_1}{(2\pi)^4}D_{(11)}(p)D_{(12)}(q)D_{(12)}(p-q)D_{(21)}(p)\\
&-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(21)}(q)D_{(21)}(p-q)D_{(11)}(p)
\end{aligned}
$$

というわけで ϕ^3 理論での 2 次のオーダまでの 11 成分の伝播関数の運動量表示は

$$
\begin{aligned}
&G_{(11)}(p)\\
&=\ D_{(11)}(p)\\
&\quad+\frac{1}{2}\Big((i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(11)}(p)D_{(11)}(q)D_{(11)}(0)D_{(11)}(p)+(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(22)}(q)D_{(22)}(0)D_{(21)}(p)\\
&\qquad-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(11)}(q)D_{(12)}(0)D_{(21)}(p)-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(11)}(q)D_{(12)}(0)D_{(21)}(p)\Big)\\
&\quad+\frac{1}{2}\Big((i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(11)}(p)D_{(11)}(q)D_{(11)}(p-q)D_{(11)}(p)+(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(22)}(q)D_{(22)}(p-q)D_{(21)}(p)\\
&\qquad-(i\lambda)^2\int\frac{d^4q}{(2\pi)^4}D_{(12)}(p)D_{(21)}(q)D_{(21)}(p-q)D_{(11)}(p)-(i\lambda)^2\int\frac{d^4q_1}{(2\pi)^4}D_{(11)}(p)D_{(12)}(q)D_{(12)}(p-q)D_{(21)}(p)\Big)
\end{aligned}
$$

となります。