積分公式

基本的な公式から応用的な公式まで無節操にのせておきます。ヒマなら自分で導いてみてください。
不定積分での積分定数は省いています。

表記

a, b は定数

太字は 3 次元ベクトル

自然対数 $\log$

絶対値 $|\ |$

逆三角関数 $\arcsin, \arccos, \arctan$

ガンマ関数 $\Gamma(n)$

- x^n の積分

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \qquad (n \neq -1)$$

$$\int \frac{1}{x} dx = \log|x|$$

$$\int \frac{1}{x^n} dx = -\frac{1}{(n-1)}\frac{1}{x^{n-1}} \qquad (n \neq 1)$$

- $(ax+b)^{-n}$ の積分

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\log|ax+b|$$

$$\int \frac{1}{(ax+b)^n} dx = -\frac{1}{a(n-1)}\frac{1}{(ax+b)^{n-1}}$$

$$\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2}\log|ax+b|$$

$$\int \frac{x^2}{ax+b} dx = \frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2}{a^3}\log|ax+b|$$

$$\int \frac{x}{(ax+b)^2} dx = \frac{1}{a^2}(\frac{b}{ax+b} + \log|ax+b|)$$

$$\int \frac{x^2}{(ax+b)^2} dx = \frac{x}{a^2} - \frac{b^2}{a^3}\frac{1}{ax+b} - \frac{2b}{a^3}\log|ax+b|$$

- $x^2 \pm a^2$ の積分

$$\int \sqrt{x^2+a^2} dx = \frac{1}{2}\big(x\sqrt{x^2+a^2} + a^2\log|x+\sqrt{x^2+a^2}|\big)$$

$$\int x\sqrt{x^2+a^2}dx = \frac{1}{3}(x^2+a^2)^{3/2}$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}dx = \frac{1}{2}\log|x^2+a^2|$$

$$\int \frac{1}{x(x^2+a^2)}dx = \frac{1}{2a^2}\log\frac{x^2}{a^2+x^2}$$

$$\int \frac{1}{x^2(x^2+a^2)}dx = -\frac{1}{a^2x} - \frac{1}{a^3}\arctan\frac{x}{a}$$

$$\int \frac{1}{(x^2+a^2)^2}dx = \frac{1}{2a^2}\frac{x}{x^2+a^2} + \frac{1}{2a^3}\arctan\frac{x}{a}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}(x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a})$$

$$\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}(a^2-x^2)^{\frac{3}{2}}$$

$$\int \frac{1}{a^2-x^2}dx = -\frac{1}{2a}\log|\frac{x-a}{x+a}|$$

$$\int \frac{x}{a^2-x^2}dx = -\frac{1}{2}\log|a^2-x^2|$$

$$\int \frac{1}{x(a^2-x^2)}dx = \frac{1}{2a^2}\log|\frac{x^2}{a^2-x^2}|$$

$$\int \frac{1}{x^2(a^2-x^2)}dx = -\frac{1}{a^2x} + \frac{1}{2a^3}\log|\frac{a+x}{a-x}|$$

$$\int \frac{1}{(a^2-x^2)^2}dx = \frac{1}{2a^2}\frac{x}{a^2-x^2} - \frac{1}{4a^3}\log|\frac{x-a}{x+a}|$$

$$\int \frac{1}{(a^2\pm x^2)^{n+1}}dx = \frac{x}{2a^2n(a^2\pm x^2)^n} + \frac{2n-1}{2a^2n}\int\frac{dx}{(a^2\pm x^2)^n} \qquad (n\neq 0)$$

$$\int \frac{x}{(a^2\pm x^2)^{n+1}}dx = \mp\frac{1}{2n(a^2\pm x^2)^n} \qquad (n\neq 0)$$

- 指数関数の積分

$$\int e^{ax}dx = \frac{1}{a}e^{ax}$$

$$\int a^x dx = \frac{a^x}{\log a} \qquad (a^x = \exp[x\log a])$$

$$\int xe^{ax}dx = (\frac{x}{a} - \frac{1}{a^2})e^{ax}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax}$$

$$\int x^3 e^{ax} dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right) e^{ax}$$

$$\int \frac{1}{e^x + 1} dx = x - \log[1 + e^x]$$

- 対数の積分

$$\int \log x \ dx = x \log x - x$$

$$\int x^n \log x \ dx = \frac{x^{n+1}}{n+1} \log x - \frac{x^{n+1}}{(n+1)^2} \qquad (n \neq -1)$$

$$\int \frac{\log x}{x} dx = \frac{1}{2} (\log x)^2$$

$$\int \frac{\log x}{\sqrt{x}} dx = 2\sqrt{x} \log x - 4\sqrt{x}$$

$$\int \log[ax + b] dx = \frac{1}{a}(ax + b)\big(\log[ax + b] - 1\big)$$

$$\int \log[x^2 + a^2] dx = x \log[x^2 + a^2] - 2x + 2a \arctan \frac{x}{a}$$

$$\int x \log[x^2 + a^2] dx = \frac{1}{2}(x^2 + a^2) \log[x^2 + a^2] - \frac{1}{2} x^2$$

- 三角関数の積分

$$\int \sin x \ dx = -\cos x$$

$$\int \sin^2 x \ dx = \frac{1}{2} x - \frac{1}{4} \sin 2x$$

$$\int \sin^3 x \ dx = -\cos x + \frac{1}{3} \cos^3 x$$

$$\int \cos x \ dx = \sin x$$

$$\int \cos^2 x \ dx = \frac{1}{2} x + \frac{1}{4} \sin 2x$$

$$\int \cos^3 x \ dx = \sin x - \frac{1}{3} \sin^3 x$$

$$\int \tan x \ dx = -\log|\cos x|$$

$$\int \tan^2 x \ dx = \tan x - x$$

$$\int \tan^3 x\ dx = \frac{1}{2}\tan^2 x + \log|\cos x|$$

$$\int \frac{1}{\sin x} dx = \log|\tan\frac{x}{2}| = \frac{1}{2}\log\frac{1-\cos x}{1+\cos x}$$

$$\int \frac{1}{\sin^2 x} dx = -\frac{1}{\tan x}$$

$$\int \frac{1}{\cos x} dx = \log|\tan(\frac{x}{2}+\frac{\pi}{4})| = \frac{1}{2}\log\frac{1+\sin x}{1-\sin x}$$

$$\int \frac{1}{\cos^2 x} dx = \tan x$$

$$\int \frac{1}{\tan x} dx = \log|\sin x|$$

$$\int \frac{1}{\tan^2 x} dx = -\frac{1}{\tan x} - x$$

$$\int \sin x \cos x\ dx = \frac{1}{2}\sin^2 x$$

$$\int \sin ax \cos bx\ dx = -\frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}$$

$$\int \frac{1}{\sin x \cos x} dx = \log|\tan x|$$

- 逆三角関数の積分

$$\int \arcsin x\ dx = x\arcsin x + \sqrt{1-x^2}$$

$$\int \arccos x\ dx = x\arccos x - \sqrt{1-x^2}$$

$$\int \arctan x\ dx = x\arctan x - \frac{1}{2}\log(1+x^2)$$

- 双曲線関数の積分

$$\int \sinh x\ dx = \cosh x$$

$$\int \cosh x\ dx = \sinh x$$

$$\int \sinh x \cosh x\ dx = \frac{1}{2}\cosh^2 x = \frac{1}{2}\sinh^2 x$$

- 指数関数と三角関数の積分

$$\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

- 指数関数の定積分 $(a > 0)$

$$\int_0^\infty e^{-ax}dx = \frac{1}{a}$$

$$\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}} \qquad (n > 0)$$

$$\int_0^\infty e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{2a}$$

$$\int_0^\infty \sqrt{x}e^{-ax}dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty \frac{x}{e^{ax}-1}dx = \frac{1}{6}(\frac{\pi}{a})^2$$

$$\int_0^\infty \frac{x}{e^{ax}+1}dx = \frac{1}{12}(\frac{\pi}{a})^2$$

$$\int_0^\infty \frac{x^3}{e^{ax}-1}dx = \frac{1}{15}(\frac{\pi}{a})^4$$

$$\int_0^\infty \frac{x^5}{e^{ax}-1}dx = \frac{8}{63}(\frac{\pi}{a})^6$$

$$\int_{-\infty}^\infty e^{-ax^2}dx = \sqrt{\frac{\pi}{a}} \qquad (\text{ガウス積分})$$

$$\int_{-\infty}^\infty xe^{-ax^2}dx = 0$$

$$\int_{-\infty}^\infty x^2e^{-ax^2}dx = \sqrt{\frac{\pi}{4a^3}}$$

$$\int_{-\infty}^\infty x^{2n}e^{-a^2x^2}dx = \frac{(2n-1)!!}{2^n}\sqrt{\frac{\pi}{a^{4n+2}}} \qquad (n!! = n\cdot(n-2)\cdot(n-4)\cdots)$$

$$\int_{-\infty}^\infty x^{2n+1}e^{-a^2x^2}dx = 0$$

$$\int_{-\infty}^\infty e^{-a^2x^2+bx}dx = \frac{\sqrt{\pi}}{a}\exp[\frac{b^2}{4a^2}]$$

$$\int_{-\infty}^\infty e^{-a^2x^2+ibx}dx = \frac{\sqrt{\pi}}{a}\exp[-\frac{b^2}{4a^2}]$$

- 積分の絶対値

$$\Big|\int_a^b f(x)\ dx\Big| \leq \int_a^b |f(x)|\ dx$$

- 積分の微分

$$\frac{d}{dx}\int_a^x f(y)dy = f(x)$$

$$\frac{d}{dx}\int_{v(x)}^{u(x)} f(y)dy = f(u(x))\frac{du}{dx} - f(v(x))\frac{dv}{dx}$$

$$\frac{d}{dx}\int_a^x f(x,y)dy = f(x,x) + \int_a^x \frac{\partial}{\partial x}f(x,y)\ dy$$

$$\frac{d}{dx}\int_{v(x)}^{u(x)} f(x,y)dy = f(x,u(x))\frac{du}{dx} - f(x,v(x))\frac{dv}{dx} + \int_{v(x)}^{u(x)} \frac{\partial}{\partial x}f(x,y)\ dy$$

- 線積分、面積分、体積積分
  - グリーンの定理 : $\oint_\gamma (F(x,y)dy + G(x,y)dx) = \int_S dxdy(\frac{\partial G(x,y)}{\partial x} - \frac{\partial F(x,y)}{\partial y})$
    γ は xy 平面上の 2 次元面 S を囲む閉曲線。
  - ストークスの定理 : $\int_S (\nabla \times \boldsymbol{A}(\boldsymbol{x})) \cdot \boldsymbol{n} ds = \oint_C \boldsymbol{A}(\boldsymbol{x}) \cdot d\boldsymbol{x}$
    S は 2 次元面、C は S を囲む閉曲線、$\boldsymbol{n}$ は C に対して右ねじの方向。
  - ガウスの発散定理 : $\int_V \nabla \cdot \boldsymbol{A}(\boldsymbol{x}) d^3x = \int_S \boldsymbol{A}(\boldsymbol{x}) \cdot \boldsymbol{n}\ dS$
    V は 3 次元領域、S はそれを囲む閉曲面、$\boldsymbol{n}$ は閉曲面 S の単位法線ベクトル (外向きを正)。